2025 MAA AMC 10B

Problem 1

The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

一袋重量为 350 克的咖啡豆的使用说明上写着，冲泡一大杯手冲咖啡需要 20 克咖啡豆．问依照标准，用这袋咖啡豆最多可以冲泡多少杯大杯咖啡?

（A） 16 **（B）** 17 **（C）** 18 **（D）** 19 **（E）** 20

Problem 2

Jerry wrote down the ones digit of each of the first 2025 positive squares: 1,4,9,6,5,6, ... What is the sum of all the numbers Jerry wrote down?

Jerry 写下了前 2025 个正整数平方的个位数字：1，4，9，6，5，6，….问 Jerry 写下的所有数的总和是多少?

（A） 9025 **（B）** 9070 **（C）** 9090 **（D）** 9115 **（E）** 9160

Problem 3

A Pascal-like triangle has 10 as the top row and 10 followed by 1 as the second row. In each subsequent row the first number is 10, the last number is 1, and, as in the standard Pascal's Triangle, each other number in the row is the sum of the two numbers directly above it. The first four rows are shown below. What is the sum of the digits of the sum of the numbers in the 11th row?

在一个三角形数阵中，第一行为 10，第二行为 10 和 1．之后的每一行，第一个数是 10，最后一个数是 1，且如同标准的杨辉三角形，中间的其他每个数都是其上方两个数的和．下图显示了数阵的前四行．考虑第 11 行中所有数的总和，问这个总和的各位数字之和是多少?

10

10 1

10 11 1

10 21 12 1

(A) 11 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 17

Problem 4

The value of the two-digit number $\underline{a}\,\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\,\underline{a}$ in base nine. What is $a+b$?

七进制的两位数 $\underline{a}\,\underline{b}$ 的值等于九进制的两位数 $\underline{b}\,\underline{a}$ 的值．问 $a+b$ 是多少?

(A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 14

Problem 5

In $\triangle ABC$, $AB=10$, $AC=18$, and $\angle B=130^\circ$.Let O be the center of the circle containing A, B , and C . What is the degree measure of $\angle CAO$?

在 $\triangle ABC$ 中，$AB=10$，$AC=18$，且 $\angle B=130^\circ$．设 O 为经过 A，B，C的圆的圆心．问 $\angle CAO$的度数是多少?

(A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

Problem 6

The line $y=\frac{1}{3}x+1$ divides the square region defined by $0\le x\le 2$ and $0\le y\le 2$ into an upper and lower region. The line $x=a$ divides the lower region into two regions of equal area. Then a can be written as $\sqrt{s}-t$, where s and t are positive integers. What is $s+t$?

直线 $y=\frac{1}{3}x+1$ 把由 $0\le x\le 2$ 和 $0\le y\le 2$ 定义的正方形区域分割成上方区域和下方区域．直线 $x=a$ 将下方区域分成两个面积相等的区域．已知 a 可以写成 $\sqrt{s}-t$ 的形式，其中 s 和 t 是正整数．问 $s+t$ 是多少?

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

Problem 7

Frances stands 15 meters directly south of a locked gate in a fence that runs east-west. Immediately behind the fence is a box of chocolates, located x meters east of the locked gate. An unlocked gate lies 9 meters east of the box, and another unlocked gate lies 8 meters west of the locked gate. Frances can reach the box by walking toward an unlocked gate, passing through it, and walking toward the box. It happens that the total distance Frances would travel is the same via either unlocked gate. What is the value of x?

一道东西走向的栅栏上有一扇上锁的门,Frances 站在上锁的门正南方 15 米处.在位于上锁的门以东 x 米处，紧贴着栅栏的后面有一盒巧克力．在巧克力盒以东 9 米处有一扇没锁的门，且在上锁的门以西 8 米处还有另一扇没锁的门．Frances 可以走向一扇没锁的门，穿过它，然后再走向巧克力盒．她通过任何一扇没锁的门走到巧克力盒的总路程是相同的．问 x 的值是多少?

(A) $3\frac{2}{7}$ **(B)** $3\frac{3}{7}$ **(C)** $3\frac{4}{7}$ **(D)** $3\frac{5}{7}$ **(E)** $3\frac{6}{7}$

Problem 8

Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$\underline{A}\,\underline{B}\,\underline{B}.\,\underline{B}\,\underline{A}$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A + B$?

Emmy 对 Max 说:“我今天订了 36 件数学社的运动衫.” Max 问:“每件运动衫多少钱?” Emmy 回答说:“我给你一个提示．总成本是 $\$\underline{A}\,\underline{B}\,\underline{B}.\,\underline{B}\,\underline{A}$，其中 A 和 B 是数字且 $A \neq 0$．” 停顿了一下后，Max 说:“这个价格很不错．”问 $A + B$ 的值是?

(A) 7 **(B)** 8 **(C)** 11 **(D)** 14 **(E)** 15

Problem 9

How many ordered triples of integers (x, y, z) satisfy the following system of inequalities?

满足以下不等式组的三元有序整数组 (x, y, z) 有多少个?

$$-x-y-z \leq -2$$

$$-x+y+z \leq 2$$

$$x-y+z \leq 2$$

$$x+y-z \leq 2$$

(A) 4 **(B)** 8 **(C)** 11 **(D)** 15 **(E)** 17

Problem 10

Let $f(n) = n^3 - 5n^2 + 2n + 8$ and $g(n) = n^3 - 6n^2 + 5n + 12$. What is the sum of all integers n such that $\frac{f(n)}{g(n)}$ is an integer?

设 $f(n) = n^3 - 5n^2 + 2n + 8$，且设 $g(n) = n^3 - 6n^2 + 5n + 12$. 使得 $\frac{f(n)}{g(n)}$ 为整数的整数 n的和是多少?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Problem 11

On Monday, 6 students went to the tutoring center at the same time, and each one was randomly assigned to one of the 6 tutors on duty. On Tuesday, the same 6 students showed up, the same 6 tutors were on duty, and the students were again randomly assigned to the tutors. What is the probability that exactly 2 students met with the same tutor both Monday and Tuesday?

星期一，有 6 名学生同时前往辅导中心，每名学生被随机分配给值班的 6 名辅导老师中的一位. 星期二，这 6 名学生又去了，值班的还是这 6 名辅导老师，学生们再次被随机分配给这些辅导老师. 问恰好有 2 名学生在星期一和星期二遇到相同的辅导老师的概率是多少?

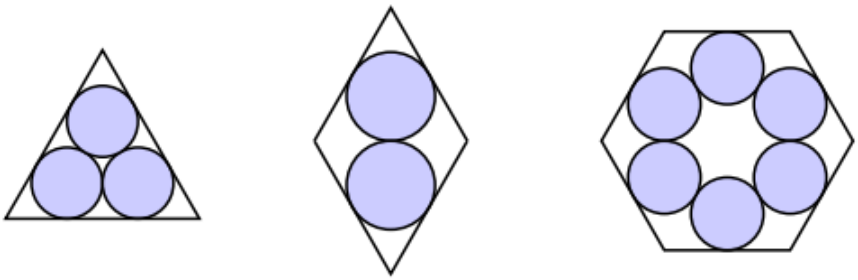
(A) $\frac{1}{16}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{1}{2}$

Problem 12

The figure below shows an equilateral triangle, a rhombus with a 60° angle, and a regular hexagon, each of them containing some mutually tangent congruent disks. Let T, R, and H, respectively, denote the ratio in each case of

the total area of the disks to the area of the enclosing polygon. Which of the following is true?

下图展示了等边三角形，有一个角是 60° 的菱形，以及正六边形，其中每个图形都包含一些相互外切的全等圆盘．设 T，R ，H 分别表示在每种情况下圆盘总面积与外框多边形的面积之比．问以下哪个选项是正确的？



(A) $T=H=R$ **(B)** $H<R=T$ **(C)** $H=R<T$ **(D)** $H<R<T$ **(E)** $H<T<R$

Problem 13

The altitude to the hypotenuse of a $30-60-90^\circ$ right triangle is divided into two segments of lengths $x<y$ by the median to the shortest side of the triangle. What is the ratio $\frac{x}{x+y}$?

在一个 $30-60-90^\circ$ 的直角三角形中，斜边上的高被该三角形较短直角边上的中线分成两条线段，长度分别为 $x<y$．问比值 $\frac{x}{x+y}$ 是多少？

(A) $\frac{3}{7}$ **(B)** $\frac{\sqrt{3}}{4}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{11}$ **(E)** $\frac{4\sqrt{3}}{15}$

Problem 14

Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

九名身高各不相同的运动员参加篮球队选拔．他们依次从装有 3 条蓝色腕带，3 条红色腕带，3 条绿色腕带的袋子中随机抽取一条腕带，且抽取后不放回．随后

他们按腕带颜色被分成蓝队，红队，绿队．每队中身高最高的成员被任命为队长．问三名队长是三名最高的运动员的概率是多少?

(A) $\frac{2}{9}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{9}{28}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

Problem 15

The sum

$$\sum_{k=1}^{\infty} \frac{1}{k^3+6k^2+8k}$$

can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$?

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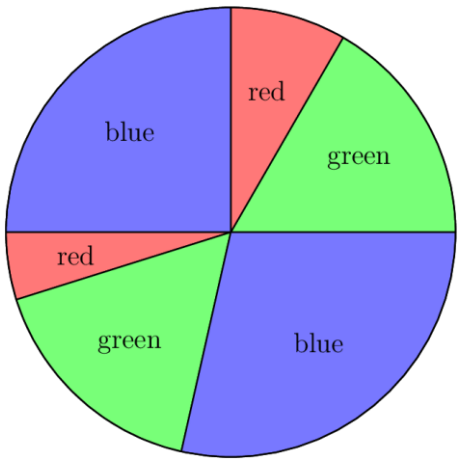
$$\sum_{k=1}^{\infty} \frac{1}{k^3+6k^2+8k}$$

的计算结果可以表示为 $\frac{a}{b}$，其中 a 和 b 是互质的正整数．问 $a+b$ 是多少?

(A) 89 **(B)** 97 **(C)** 102 **(D)** 107 **(E)** 129

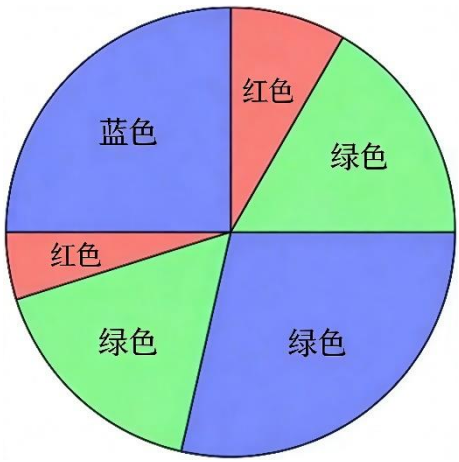
Problem 16

A circle has been divided into 6 sectors of different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

一个圆被分成了 6 个大小不同的扇形．然后将其中 2 个扇形涂成红色，2 个涂成绿色，2 个涂成蓝色，使得任意两个相邻的扇形颜色都不同．下图展示了一种这样的着色方式．



问共有多少种不同的着色方式?

（A）12　　（B）16　　（C）18　　（D）24　　（E）28

Problem 17

Consider a decreasing sequence of n positive integers $x_1 > x_2 > \cdots > x_n$ that satisfies the following conditions:

- The average of the first 3 terms in the sequence is 2025.
- For all $4 \leq k \leq n$, the average of the first k terms is 1 less than the average of the first $k-1$ terms.

What is the greatest possible value of n?

考虑由 n 个正整数组成的递减数列 $x_1 > x_2 > \cdots > x_n$，它满足以下条件：

- 数列前 3 项的平均值是 2025．
- 对于所有 $4 \leq k \leq n$，前 k 项的平均值比前 $k-1$ 项的平均值小 1．

问 n 的最大可能值是多少?

（A）1013　　（B）1014　　（C）1016　　（D）2016　　（E）2025

Problem 18

What is the ones digit of the sum

$$\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2025}\rfloor?$$

(Recall that $\lfloor x\rfloor$ represents the greatest integer less than or equal to x.)

下面算式和的个位数字是多少?

$$\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2025}\rfloor$$

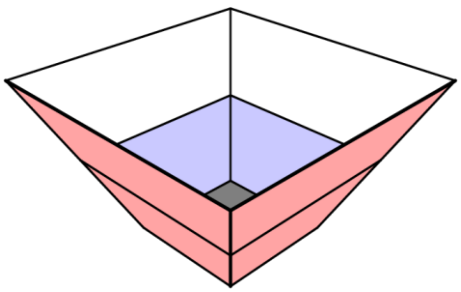
（注意：$\lfloor x\rfloor$ 表示小于或等于 x 的最大整数.）

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

Problem 19

A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids. How many more minutes will it take to fill the remainder of the container?

如图所示，容器的底部是 1×1 的正方形，顶部是 3×3 的开放正方形，还有四个全等的梯形侧面.开始时容器是空的,用一根水管以恒定速率向容器中注水,需要 35 分钟才能将水注满至梯形侧面中线的高度. 问还需要多少分钟才能将容器的剩余部分注满?



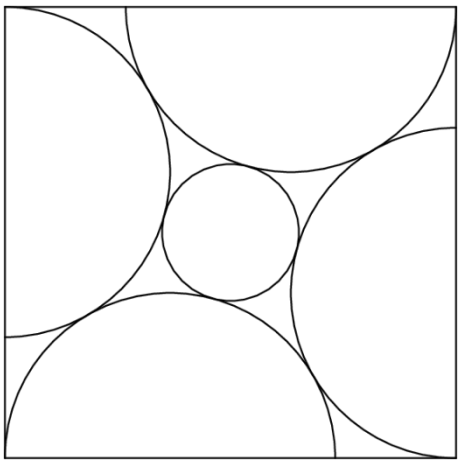
(A) 79 **(B)** 85 **(C)** 90 **(D)** 95 **(E)** 105

Problem 20

Four congruent semicircles are inscribed in a square of side length 1 so that their diameters are on the sides of the square, one endpoint of each diameter is

at a vertex of the square, and adjacent semicircles are tangent to each other. A small circle centered at the center of the square is tangent to each of the four semicircles, as shown below. The diameter of the small circle can be written as $\left(\sqrt{a}+b\right)\left(\sqrt{c}+d\right)$, where $a,\ b,\ c$, and d are integers. What is $a+b+c+d$?

四个全等的半圆内接于边长为 1 的正方形中，它们的直径在正方形的边上，相邻的半圆彼此相切，且每个直径的一个端点位于正方形的顶点．如下图所示，一个小圆的圆心位于正方形的中心，并与这四个半圆都相切．这个小圆的直径可以写成 $\left(\sqrt{a}+b\right)\left(\sqrt{c}+d\right)$ 的形式，其中 a，b，c，d 都是正整数．问 $a+b+c+d$ 是多少？



（A）3　　**（B）**5　　**（C）**8　　**（D）**9　　**（E）**11

Problem 21

Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

对于 3×3 方格表中的 9 个方格，每个方格都要涂上红色、蓝色或黄色，且需满足以下条件：每个红色方格都与至少一个蓝色方格有公共边，每个蓝色方格都与至少一个黄色方格有公共边，每个黄色方格都与至少一个红色方格有公共边．通过旋转、反射或者它们的组合可以互相得到的涂色方式视为相同的，问共有多少种不同的涂色方式？

(A) 3 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 27

Problem 22

A seven-digit positive integer is chosen at random. What is the probability that the number is divisible by 11, given that the sum of its digits is 61?

随机选取一个七位正整数，已知其各位数字之和为 61，问这个数能被 11 整除的概率是多少?

(A) $\frac{3}{14}$ **(B)** $\frac{3}{11}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{4}{11}$ **(E)** $\frac{3}{7}$

Problem 23

A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \cdot 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

考虑有 141 行与 91 列的矩形方格表，每个方格中可填入两个数．Horace 与 Vera 分别将从 1 到 $141 \cdot 91 = 12{,}831$ 的所有数填入方格中．Horace 沿水平方向填写：他在第 1 行从左到右依序填入 1 至 91，在第 2 行从左到右依序填入 92 至 182，并以此类推直至填完第 141 行．Vera 沿垂直方向填写：她在第 1 列从上到下依序填入 1 至 141，在第 2 列从上到下依序填入 142 至 282，并以此类推直至填完第 91 列．问有多少个方格中两人填入的数相同?

(A) 7 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 19

Problem 24

A frog hops along the number line according to the following rules.

- It starts at 0.

- If it is at 0, then it moves to 1 with probability $\frac{1}{2}$ and it disappears with probability $\frac{1}{2}$.
- For $n = 1$, 2 or 3, if it is at n, then it moves to $n + 1$ with probability $\frac{1}{4}$, it moves to $n - 1$ with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.

What is the probability that the frog reaches 4?

一只青蛙按照如下规则在数轴上跳跃:

- 它从 0 出发。
- 若它在 0 处，以 $\frac{1}{2}$ 的概率跳到 1，以 $\frac{1}{2}$ 的概率消失。
- 当$n = 1$，2，3，若它在 n 处，则以 $\frac{1}{4}$ 的概率跳到 $n + 1$；以 $\frac{1}{4}$ 的概率跳到 $n - 1$；以 $\frac{1}{2}$ 的概率跳到消失;

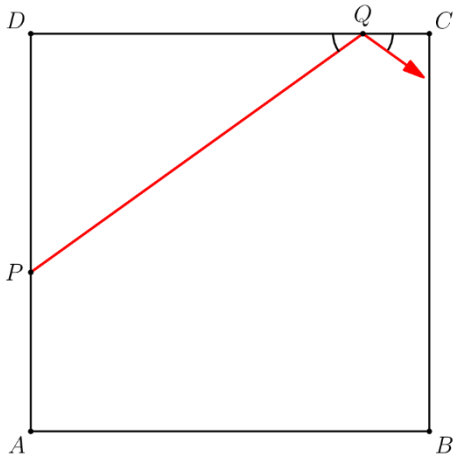
问青蛙最终到达 4 的概率是多少?

(A) $\frac{1}{101}$ **(B)** $\frac{1}{100}$ **(C)** $\frac{1}{99}$ **(D)** $\frac{1}{98}$ **(E)** $\frac{1}{97}$

Problem 25

Square $ABCD$ has sides of length 4. Points P and Q lie on $\overline{AD}$ and $\overline{CD}$, respectively, with $AP = \frac{8}{5}$ and $DQ = \frac{10}{3}$. A path begins along the segment from P to Q and continues by reflecting against the sides of $ABCD$ (with congruent incoming and outgoing angles). If the path hits a vertex of the square, it terminates there; otherwise it continues forever. At which vertex does the path terminate?

正方形 $ABCD$ 的边长为 4，点 P 与点 Q 分别位于 $\overline{AD}$ 与 $\overline{CD}$ 上，且 $AP = \frac{8}{5}$，$DQ = \frac{10}{3}$. 如图所示，一条路径以线段 PQ 为起始，并在触及正方形 $ABCD$ 的边时反射前进（入射角与反射角相等）. 若路径碰到正方形的顶点，则在此终止；否则会继续下去，不断反射. 问这条路径最终会在哪个顶点终止?



(A) A　　**(B)** B　　**(C)** C　　**(D)** D

(E) The path continues forever. | 这条路径无限延伸

2025 AMC 10B Answer Key												
题目	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
答案	B	D	D	A	C	C	C	C	C	A	B	C	A
题目	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
答案	C	D	D	B	D	D	A	C	A	C	E	B	

2025 AMC10B Solution

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